# 16.1 Square Root Property

## Introduction to Quadratic Equations

A **quadratic equation** is any equation that can be written in the standard form

$$ax^2 + bx + c = 0$$

where *a*,*b*, and *c* are real numbers and $a > 0$. In other words, a quadratic equation is any equation with a polynomial of degree 2 ( $x^2$ term). A quadratic equation may have up to 2 real solutions.

***Sample Problem:*** Write the quadratic equation in standard form and label *a*,*b*,*c*.

$$3x^2 = 5x - 7$$

***Solution:*** The standard form of a quadratic equation must have all terms on one side, and must have $a > 0$. We must move everything to one side.

$$3x^2 = 5x - 7$$

$$3x^2 - 5x + 7 = 0$$

Thus $a = 3$ , $b = -5$ , $c = 7$

***Student Practice: Write each quadratic equation in standard form and label a, b, c.***

1. $x^2 + 8x = 2$
2. $x(x+5) = 4x - 3$
3. $-2x^2 + x + 9 = 0$

We have solved quadratic equations before using the zero factor property. An example is shown below. However, we will now learn that this method cannot always be used.

***Sample Problem:*** ***Solve for x.*** $x^2 - 5x = 14$

***Solution:*** We must first write the equation in standard form.

$$x^2 - 5x = 14$$

$$x^2 - 5x - 14 = 0$$

$$(x-7)(x+2) = 0$$

$$x - 7 = 0 \qquad x + 2 = 0$$

$$x = 7 \qquad x = -2$$

***Check:***

$$(7)^2 - 5(7) = 14$$

$$49 - 35 = 14$$

$$14 = 14$$

A quadratic equation in which the $x$ term is missing ( or $b = 0$ ) can be easily solved using the **principle of square roots**.

**PRINCIPLE OF SQUARE ROOTS**

If $x^2 = c$, the solutions are $x = \pm\sqrt{c}$.

If $(x-a)^2 = c$, the solutions are $x = a \pm \sqrt{c}$

***Sample Problem:*** ***Solve for x*** a. $x^2 - 2 = 27$ b. $(x-4)^2 = 18$

***Solution:*** ***a.*** We first isolate $x^2$, then apply principle.

$$x^2 - 2 = 27$$
$$x^2 = 28$$
$$x = \pm\sqrt{28}$$
$$x = \pm 2\sqrt{7}$$

**b.** Apply the square root property.

$$(x-4)^2 = 18$$
$$x - 4 = \pm\sqrt{18}$$
$$x = 4 \pm \sqrt{18}$$
$$x = 4 \pm 3\sqrt{2}$$

***Student Practice: Solve each equation for x.***

4. $x^2 = 81$

5. $3x^2 = 12$

6. $x^2 = 18$

7. $5x^2 + 10 = 15$

8. $(x-3)^2 = 36$

9. $(x+7)^2 = 4$

10. $(x+4)^2 = 11$

11. $4(x+3)^2 = 24$

## 16.2 Completing The Square

The idea behind the completing the square method is to modify an equation that is otherwise unfactorable so that the square root principle can be used. Here are the steps to follow for completing the square:

***Sample Problem:: Solve the equation*** $x^2 + 10x - 3 = 0$

**Step 1:** If $a \neq 1$, first divide the equation by *a* (the number in front of $x^2$).

$$x^2 + 10x - 3 = 0$$

**Step 2:** Move the constant term to one side. Get the variables alone on one side. **(Note: In this method, we do NOT put the equation in standard form)**

$$x^2 + 10x = 3$$

**Step 3:** We now divide *b* by 2, square this, and add this to both sides. This will "complete the square", hence the name. Since $\left(\frac{10}{2}\right)^2 = 25$, we add this to both sides.

$$x^2 + 10x + 25 = 3 + 25$$

or

$$x^2 + 10x + 25 = 28$$

**Step 4:** Factor the side with the quadratic term. Note that this will always factor into a perfect square.

$$(x+5)(x+5) = 28$$

or

$$(x+5)^2 = 28$$

**Step 5:** Use the square root property to solve the equation.

$$(x+5)^2 = 28$$

$$x + 5 = \pm\sqrt{28}$$

$$x = -5 \pm \sqrt{28}$$

$$x = -5 \pm 2\sqrt{7}$$

The solutions are $x = -5 + 2\sqrt{7}, x = -5 - 2\sqrt{7}$ .

***Student Practice: Solve each equation using the <u>completing the square method</u>.***

1. $x^2 + 6x - 2 = 0$

2. $x^2 + 2x = 17$

3. $x^2 - 6x - 16 = 0$

4. $2x^2 - 8x - 6 = 0$

5. $x^2 + 7x - 3 = 0$

6. $x^2 - 3x = 9$

# 16.3 The Quadratic Formula

The **quadratic formula** is the most commonly used tool for solving quadratic equations because it can be used to solve **ANY** quadratic equation, whether or not it is factorable (hence the name "quadratic formula"). The only condition is that the equation be in standard form (move everything to one side with a 0 on the other), and that you simplify your result.

**<u>THE QUADRATIC FORMULA</u>**

The solutions of any quadratic equation in standard form

$$ax^2 + bx + c = 0$$

are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where $a$ is the coefficient of $x^2$, $b$ is the coefficient of $x$, and $c$ is the constant.
***(NOTE: EQUATION MUST BE IN STANDARD FORM)***

***Sample Problem:*** ***Solve for x*** $x^2 - 6x = 3$

***Solution:*** First note that the equation must be written in standard form, so subtract 3.

$$x^2 - 6x = 3$$
$$x^2 - 6x - 3 = 0$$

Now note that $a = 1$, $b = -6$, and $c = -3$.
Use these values in the formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-3)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{36 + 12}}{2}$$

$$x = \frac{6 \pm \sqrt{48}}{2}$$

$$x = \frac{6 \pm 4\sqrt{3}}{2}$$

Now we must simplify:

$$x = \frac{2(3 \pm 2\sqrt{3})}{2}$$

$$x = 3 \pm 2\sqrt{3}$$

The two answers are then

$x = 3 + 2\sqrt{3}$ 
$x = 3 - 2\sqrt{3}$

***OR***

$x \approx 6.46$
$x \approx -0.46$

***Student Practice: Solve each equation for x using the quadratic formula.***

1. $x^2 + 2x = 17$

2. $x^2 - 3x = 9$

3. $3x^2 = 4x + 2$

4. $2x(x - 3) = -1$